RADIATIVELY COOLING FLOWS OF SELF-GRAVITATING FILAMENTARY CLOUDS

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Abstract

We study the dynamics of a self-gravitating cooling filamentary cloud using a simplified model. We concentrate on the radial distribution and restrict ourselves to quasi-hydrostatic cylindrical symmetric cooling flows. For a power-law dependence of cooling function on the temperature, similarity solution is found. We consider obtically thin filaments with constant mass per unit length and the solutions are parameterized by their line masses. There is no polytropic relation between the density and the pressure. The filament experiences radiative condensation, irrespective of the γ, the gas specific heat ratio. So, due to the quasi-hydrostatic flows the filament becomes denser and the density at the center (ρ_c) is proportional to $(t_0 - t)^{-1}$. We also found that the radius of the filament (r_c) decreases in proportion to $(t_0 - t)^{\frac{1}{2}}$.

Subject heading: hydrodynamic - instabilities - ISM: clouds - stars: formation

1 INTRODUCTION

Interstellar molecular clouds are very complex structures in which many physical processes come into play. In these clouds, rich and complex structures have been seen (see, e.g., Myers 1991; Blitz 1993). Since most star formation essentially occurs in molecular clouds, understanding the origin and evolution of their structures is very important.

Filamentary structures associated with clumps and cores are very common (e.g., Schneider & Elmegreen 1979; Houlahan & Scalo 1992; Wiseman & Adams 1994; Alves et al. 1998; Harjunpaa et al. 1999). It is now widely accepted that this hierarchical structure has been formed by gravitational instability. In this scenario for the formation of the hierarchical structure, four distinct phases have been studied: (a) gravitational instability along filamentary cloud led to the formation of clumps (e.g., Nagasawa 1987; Nakamura, Hanawa, & Nakano 1993; Gehman, Adams, & Watkins 1996; Nagai, Inutsuka, & Miyama 1998; Fiege & Pudritz 2000); (b) these clumps become geometrically thin disks perpendicular to the filament axis (Nakamura, Hanawa, & Nakano 1995; Nakamura et al. 1999); (c) The disks collapse and because of gravitational instability, each of them fragments into cloud cores (Nakamura & Hanawa 1997; Saigo & Hanawa 1998); and finally (d) these cores are scattered by gravitational interaction.

It must be noted that in this scenario, the overall collapse of filamentary cloud is slower than the growth of local density fluctuations. In fact, It has been assumed that the parent filamentary cloud is well below the critical mass per unit length required for radial collapse to a spindle. This possibility exist that filamentary cloud collapses, however, and the collapse of filaments (Bonnell & Bastien 1991; Inutsuka & Miyama 1993; Kawachi & Hanawa 1998) has begun to be studied.

Most previous work has studied collapse of spherically symmetric clouds (e.g., Larson 1972; Terebey, Shu, & Cassen 1984; Foster & Chevalier 1993). Calculations showed that the temperature of a collapsing spherical

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molecular cloud remains roughly constant in the early phases of star formation (see, e.g., Larson 1969). Following the collapse of a spherical cloud with three dimensional numerical simulation is very difficult. But there exist asymptotic similarity solution for the collapse of a spherical cloud (Larson 1969; Penston 1969; Shu 1977). However, similarity solutions for the collapse of a filamentary cloud were investigated, and different sets have been found (Inutsuka & Miyama 1992; Kawachi & Hanawa 1998; Semelin, Sanchez, & de Vega 1999). But it needs more work to be done. Kawachi & Hanawa (1998) investigated gravitational collapse of a filamentary cloud using zooming coordinates (Bouquet et al. 1985). They used a polytropic equation of state to indicate the effects of deviations from isothermality in the collapse.

In fact all of these authors followed the usual (Jeans) scenario for star formation. But as we shall show in this paper, there exists another (non-Jeans) scenario for star formation in filamentary clouds. Because of radiative cooling, a filamentary cloud which is in equilibrium, may evolve on a longer timescale. Thus, radiative condensation or expansion flows can be formed in a cooling filamentary cloud. So, although the cloud is in fact quasi-hydrostatic, we may have density collapse due to the presence of a radiative condensation flow. The same problem has been investigated for the dynamics of cooling spherical clouds (Meerson, Megged, & Tajima 1996; hereafter MMT). In the present work we study radiative flows in a filamentary cloud using a family of similarity solutions and instead of a polytropic equation of state, the energy equation is used. We shall find solutions that indicate no polytropic relation between the pressure and the density. Since we consider the constraint of a constant mass per unit length, the solutions are parameterized by their line masses. We show that for the power-law radiative cooling function, the dynamics of quasi-hydrostatic cylindrical symmetric flows in a filamentary cloud is independent of the specific heat ratio of the gas.

The plan of this paper is as follows. We present the general mathematical framework and equations in § 2. By introducing similarity solution, the dynamics of a radiatively cooling filamentary cloud have been studied in § 3. We concentrate on the quasi-hydrostatic flows. In § 4 after discussing about the behaviour of centeral density and the radius of the cloud, we conclude with the implication of our model.

2 GENERAL FORMULATION

We present here the basic equations used to describe cylindrically symmetric radiatively cooling flows. In this work, we neglect effects of viscosity and magnetic fields. In cylindrical coordinates (r, φ, z), which the filamentary cloud is assumed to be long in the $z-$direction, we have:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v) = 0, \tag{1}$$

$$\rho(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}) + \frac{\partial p}{\partial r} + \rho\frac{\partial \Psi}{\partial r} = 0, \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Psi}{\partial r}) = 4\pi G\rho, \tag{3}$$

where ρ, v, p, Ψ denote the gas density, radial velocity, pressure, and gravitational potential, respectively. We write energy equation and equation of state as:

$$\frac{1}{\gamma - 1}(\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial r}) + \frac{\gamma}{\gamma - 1}\frac{p}{r}\frac{\partial}{\partial r}(rv) + \Lambda(\rho, T) = 0, \tag{4}$$

$$p = \frac{R_g}{\mu_g}\rho T, \tag{5}$$

where $\Lambda(\rho, T)$ is the radiative loss function and R_g and μ_g are the gas constant and the effective molar mass. Neglecting any external heating, we can fit the radiative loss function by a power law,

$$\Lambda(\rho, T) = A_\nu \rho^2 T^\nu, \tag{6}$$

where ν and A_ν are constants. Depending on the selected interval of the temperature, these parameters can be determined (Spitzer 1978).

We are interested in quasi-hydrostatic flows in filamentary clouds. In fact this situation occurs when the cooling is slow on the dynamic time scale. For quasi-hydrostatic flows equation (2) becomes

$$\frac{\partial p}{\partial r} + \rho\frac{\partial \Psi}{\partial r} \simeq 0. \tag{7}$$

The same relation have been used by MMT.

To simplify the problem, we introduce the dimensionless variables according to

$$\rho \rightarrow \hat{\rho}\rho, p \rightarrow \hat{p}p, \Psi \rightarrow \hat{\Psi}\Psi, T \rightarrow \hat{T}T, v \rightarrow \hat{v}v, r \rightarrow \hat{r}r, t \rightarrow \hat{t}t, \tag{8}$$

where

$$\hat{\rho} = \rho_0, \hat{p} = p_0, \hat{T} = T_0, \hat{\Psi} = \frac{p_0}{\rho_0}, \hat{r} = (\frac{p_0}{4\pi G\rho_0^2})^{\frac{1}{2}}, \hat{t} = \frac{p_0}{(\gamma-1)A_\nu\rho_0^2T_0^\nu}, \hat{v} = \frac{\hat{r}}{\hat{t}}. \tag{9}$$

Under the transformation equations (8), the equations (1) and (7) do not change and the rest of equations are cast into these forms:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Psi}{\partial r}) = \rho, \tag{10}$$

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial r} + \gamma\frac{p}{r}\frac{\partial}{\partial r}(rv) + \rho^{2-\nu}p^\nu = 0. \tag{11}$$

3 SIMILARITY SOLUTIONS

It is useful to study the similarity solutions of equations (1), (7), (10) and (11). We introduce the similarity variable as $y = \frac{r}{r_0(t)}$, where $r_0(t) = (t_0 - t)^{\frac{1}{2}}$ and $t < t_0$. Using this transformation, we have the following forms of ρ, p, v and Ψ:

$$\rho(r,t) = (t_0 - t)^{-1}R(y), \tag{12}$$

$$p(r,t) = (t_0 - t)^{-1}P(y), \tag{13}$$

$$v(r,t) = (t_0 - t)^{-\frac{1}{2}}V(y), \tag{14}$$

$$\Psi(r,t) = S(y), \tag{15}$$

where $R(y)$, $P(y)$, $V(y)$ and $S(y)$ satisfy in these equations:

$$R + \frac{y}{2}\frac{dR}{dy} + \frac{1}{y}\frac{d}{dy}(yRV) = 0, \tag{16}$$

$$\frac{dP}{dy} + R\frac{dS}{dy} = 0, \tag{17}$$

$$\frac{1}{y}\frac{d}{dy}(y\frac{dS}{dy}) = R, \tag{18}$$

$$P + (\frac{y}{2} + V)\frac{dP}{dy} + \gamma\frac{P}{y}\frac{d}{dy}(yV) + R^{2-\nu}P^\nu = 0. \tag{19}$$

This form of similarity solutions deals with blow-up, or at least rapid growth, in a finite time t_0. Since we are interested in cooling flows of an isolated filamentary cloud, using transformation equation (8), the constraint of a constant mass per unit length can be written as

$$\int_0^\infty \rho r dr = m = const, \tag{20}$$

where $m = \frac{2G\rho_0}{p_0}M$ and M is mass per unit length.

Now we have a set of ordinary differential equations. It is interesting that equations (16), (17), (18) and (19) are integrable. Using equation (16) we obtain

$$yRV + \frac{1}{2}y^2R = c, \tag{21}$$

where c is a constant. We may consider flows in which on the axis of the filamentary cloud the velocity to be zero. So we obtain

$$V(y) = -\frac{y}{2}. \tag{22}$$

It shows that the velocity of cylindrical symmetric flow is independent of ν, the exponent of power-law cooling function. MMT showed that for spherical flows, depending on the ν we may have gas inflow or outflow. Substituting equation (22) in (19), we find

$$P(y) = (\gamma - 1)^{\alpha}[R(y)]^{\beta}, \tag{23}$$

where $\alpha = \frac{1}{\nu-1}$ and $\beta = \frac{\nu-2}{\nu-1}$. This relation shows that ν may have positive or negative values except for $\nu = 1$ or 2. In addition there is no polytropic relation between the pressure and the density.

Finally, after substituting equation (23) in equations (17) and (18) we obtain

$$\frac{1}{\xi}\frac{d}{d\xi}(\xi\frac{d\theta}{d\xi}) + \theta^{n} = 0, \tag{24}$$

where

$$\theta = [R(y)]^{\frac{1}{n}}, \xi = (n+1)^{-\frac{1}{2}}(\gamma-1)^{\frac{1}{2n}}y, n = 1-\nu. \tag{25}$$

Equation (24) represent the form of the Lane-Emden equation appropriate for cylindrical polytropes. This equation in the study of the equilibrium of gaseous filaments has been studied (Ostriker 1964). The boundery conditions are

$$\theta(\xi = 0) = 1, (\frac{d\theta}{d\xi})_{\xi=0} = 0. \tag{26}$$

It can be easily shown that solutions of the cylindrical Lane-Emden equation satisfying boundary conditions (26) must decrease monotonically from the origin and eventually approach zero. For $n = 0$ ($\nu = 1$) or $n = 1$ ($\nu = 0$), this equation becomes linear and the solutions can be readily obtained. If $\theta_n(\xi)$ reperesnt the required solution, we have

$$\theta_0(\xi) = 1 - \frac{1}{4}\xi^2, \theta_1(\xi) = J_0(\xi). \tag{27}$$

We note that in our problem $\theta_0(\xi)$ do not represent a solution. For other values of n, the solutions are not in closed forms. In general, however, Ostriker (1964) obtained a series solution:

$$\theta_n(\xi) = 1 - \frac{1}{(1!)^2 2^2}\xi^2 + \frac{n}{(2!)^2 2^4}\xi^4 - \frac{n(3n-2)}{(3!)^2 2^6}\xi^6 + \frac{n(18n^2-29n+12)}{(4!)^2 2^8}\xi^8 - \dots. \tag{28}$$

We note that the solutions must satisfy in the constraint of conservation of mass. In fact the solutions of the cylindrical Lane-Emden equation are subject to the same homology transformations as are the solutions of the spherical Lane-Emden equation: if $\theta_n(\xi)$ is a solution, then $A^{\frac{2}{n-1}}\theta_n(A\xi)$ will also be a solution ($n \neq 1$). Using this transformation it is possible after finding the solution, then rescale it. So equation (20) gives

$$A^{\frac{2n}{n-1}}\int_0^{\xi_1}[\theta_n(A\xi)]^n \xi d\xi = \frac{m}{n+1}(\gamma-1)^{\frac{1}{n}}, \tag{29}$$

where ξ_1 is the first zero of $\theta_n(\xi)$. All of the solutions $\theta_n(\xi)$ have first zero; i.e., the radii and masses of the filamentary clouds are finite. For mathematical proof of this point see Ostriker (1964). If $n = 1$ ($\nu = 0$), then $A\theta_1(\xi)$ will be a solution and equation (29) replaced by

$$A\int_0^{\xi_1}\theta_1(\xi)\xi d\xi = \frac{m}{2}(\gamma-1), \tag{30}$$

where $\theta_1(\xi) = J_0(\xi)$, so

$$A = (\gamma-1)\frac{m}{2\xi_1 J_1(\xi_1)}. \tag{31}$$

Equation (12) shows that for quasi-hydrostatic cylindrical symmetric cooling flows the density at the center (ρ_c) increases in proportion to $(t_0-t)^{-1}$. It is slower than the growth of density at the center, during gravitational collapse which is in proportion to $(t_0 - t)^{-2}$ (Kawachi & Hanawa 1998). We see that for cooling flow in the filamentary clouds ρ_c is independent of ν; But in spherical quasi-hydrostatic cooling flow ρ_c depends on ν and increases in proportion to $(t_0 - t)^{-\frac{3}{2+\nu}}$ (MMT).

Since the radius of the cloud (r_c) is defined by the first zero of $\theta_n(\xi)$, we can see that r_c decreases in proportion to $(t_0 - t)^{\frac{1}{2}}$ which is independent of ν. MMT showed that in spherical clouds r_c is proportional to $(t_0 - t)^{\frac{1}{\nu+2}}$ which depends on ν.

Another important difference between cylindrical and spherical flows exists. In fact MMT showed that the dynamics of quasi-hydrostatic spherical flows depends on γ, the gas specific ratio. They found $\gamma = \frac{4}{3}$ is a critical value, i.e., if γ is greater (smaller) than $\frac{4}{3}$, then the cloud condenses (expands). But here we showed that in quasi-hydrostatic cylindrical flows there is no critical value. In fact for all value of $\gamma > 1$, the filamentary cloud undergo radiative condensation.

4 DISCUSSION AND CONCLUSIONS

In this paper we have studied quasi-hydrostatic flows of radiatively cooling filamentary clouds. A non-Jeans scenario for star formation in these clouds have been investigated. This scenario in spherical clouds were given as far as we know for the first time by MMT.

When the line mass of a filamentary cloud exceeds the critical line mass, the entire filament collapses toward the axis without fragmentation. But a filamentary cloud which is in equilibrium is unstable along the axis and it will fragment. In this paper we suggested another possibility to this view point. A filamentary cloud which is in equilibrium may evolve on a longer time-scale, i.e., due to the cooling of the cloud quasi-hydrostatic flows can be formed. Considering a simple power-law cooling function, we showed that the filament experience radiative condensation. This cooling flow can be parameterized by the line mass of the filament.

It is remarkable that the decrement of the radius and the growth of density at the center of the cloud are independent of ν; i.e., we have $r_c \propto (t_0 - t)^{\frac{1}{2}}$ and $\rho_c \propto (t_0 - t)^{-1}$. We must note that these results strongly depend on the form of the cooling function. Since these behaviors of r_c and ρ_c obtained based on the similarity solution, it must be confirmed by numerical calculation of solutions. We can see that for other forms of cooling function, finding similarity solution is much more difficult. Thus, numerical simulations will reveal behavior of cooling flows in filaments. Accordingly, initial conditions which will led to the formation of such flows can be determined. We expect that the similarity solution for quasi-hydrostatic flows in filamentary clouds will serve as a guideline for understanding filaments and doing numerical simulations.

Each filament typically may contains several distinct clumps which are spaced along the filament. We can consider the filament and the embedded clumps as a self-gravitating system which is in equilibrium (Curry 2000). Theoretical arguments suggest that the mass of each clump is much larger than a typical stellar mass (see, e.g., Fiege & Pudritz 2000). In our scenario on a longer time-scale the mass and the shape of these clumps have been affected by quasi-hydrostatic cooling flows. In future work, it will be interesting to investigate the effects of cooling flows on the clumps.

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